S.No. 11007

M.Sc. (Mathematics), Second Semester, May 2017

Real Analysis – II

Time: 3 Hours

Marks:80

Note: Attempt five questions in all, selecting one question from each unit. Unit V is compulsory.

UNIT-I

Q. 1 State and prove Riemann's theorem

(a) on the rearrangements of terms of a series. 8

(b) Prove that convergent series of continuous functions may have a discontinuous sum. 8

Q. 2 (a) Suppose $f_n \to f$ uniformly on a set E in a matic space. Let x be a limit point of E and suppose that $\lim_{t\to x} f_n(t) = A_n, n = 1,2,3, \ldots\ldots$ then $\{A_n\}$ converges, and $\lim_{t\to x} f(t) = \lim_{n\to\infty} A_n$ 8

(b) Suppose $\{f_n\}$ is a sequence of functions defined on E, and suppose $|f_n(x)| \leq M_n, x \in E \,\&\, n = 1,2,3, \ldots\ldots$ then $\sum f_n$ converges uniformly on E if $\sum M_n$ converges. 8

UNIT-II

Q. 3 (a) Let $\sum C_n$ converges and let $f(x) = \sum_{n=0}^{\infty} C_n x^n$ $(-1 < x < 1)$. Then $\lim_{x\to 1} f(x) = \sum_{n=0}^{\infty} C_n$ 8

(b) Show that a linear operator A on a finite dimensional vector space X is one-to-one iff the range of A is all of X. 8

Q. 4 (a) Let E is an open set in IR^n, f maps E into IR^m, f is differentiable at $x_0 \in E$, g maps an open set convtaining $f(E)$ into IR^k and g is differentiable at $f(x_0)$. Then the mapping F of E into IR^k defined by $F(x) = g(f(x))$ is differentiable at x_0, and $F'(x_0) = g'(f(x_0)).f'(x_0)$. 8

(b) Suppose f maps a convex open set $E \subset \mathrm{IR}^n$ into IR^m, f is differentiable in E and there is a real number M such that $||f'(x)|| \leq M$ for ever $x \in E, then\ |f(b) - f(a)| \leq M|b - a|, \forall a, b \in E$. 8

UNIT-III

Q. 5 State and prove the inverse function theorem. 16

Q. 6 (a) If w and γ are K- and m- forms, respectively of class C' in E, then $d(w \wedge \gamma) = (dw)\wedge\gamma + (-1)^k w \wedge d\gamma$ 8

P.T.O.

(b) Suppose T is a $\mathcal{C}'$ –mapping of an open set $E \subset IR^n$ into an open set $V \subset IR^m$, Φ is a k-surface in E, and w is a k-form in V then $\int_{T\Phi} w = \int_{\Phi} w_T$ 8

UNIT-IV

Q. 7 (a) Let f be a function defined on [a,b]. Then f is of bounded variation iff it can be expressed as a difference of two monotone increasing fomations on [a,b]. 8

(b) Prove that a function f defined and absolutely continuous on [a,b] then f is of bounded variation on [a,b]. 8

Q. 8 (a) Let $1 \leq P \leq \infty$. Then for every pair $f, g \in L^P$, the following inequality holds: 8

$||f + g||_P \leq ||f||_P + ||g||_P$

(b) What is outer measure of a set? Prove that outer measure is countable subadditive. 8

UNIT-V

Q. 9 Compulsory question. (2 marks each)

i. What is the difference between pointwise convergence and uniformly convergence.

ii. Let $f_n(x) = \frac{Sin\, nx}{\sqrt{n}}$, ($x \in$ IR & n = 1,2,3, ….) then show that $\{f_n\} \nrightarrow f'$.

iii. Define finite dimensional rector space.

iv. Define $f'(x)$ in IRⁿ.

v. Prove that outermeasure of a countable set is zero.

vi. Define functions of bounded variation.

vii. If $m^*(A) = 0$, then $m^*(A \cup B) = m^*(B)$.

viii. Show that $f: [a, b] \to$IR given by :

$f(x) = \begin{cases} |if\ x\ is\ irrational \\ \infty\ if\ x\ is\ rational \end{cases}$ is in $L^\infty[a, b]$

S.No. 11026

M.Sc. (Mathematics), Second Semester, May 2017

Advanced Abstract Algebra-II

Time: 3 Hours Marks:80

Note: Attempt five questions in all, selecting one question from each unit. Unit V is compulsory. All questions carry equal marks.

UNIT-I

Q. 1 (a) If (N_i), $1 \le i \le k$, is a family of R-submodules of a module M, prove that 8

$\sum_{i=1}^{k} N_i = \{x_1 + x_2 + \cdots . +x_k | x_i \in N_i\}$

(b) Let A and B be R submodules of R-modules M and N respectively. Prove that 8

$\frac{M \times N}{A \times B} \simeq \frac{M}{A} \times \frac{N}{B}$

Q. 2 (a) State and prove Schur's Lemma. 7

(b) find the aselian group generated by $\{x_1, x_2, x_3\}$ subject to $5x_1 + 9x_2 + 5x_3 = 0$, $2x_1 + 4x_2 + 2x_3 = 0$ and $x_1 + x_2 - 3x_3 = 0$ 9

UNIT-II

Q. 3 For an R-module M. Prove that the following are equivalent: 16

(i) M is artinian.

(ii) every quotient module of M is finitely cogenerated.

(iii) Every non empty set S of submodules of M has a minimal element.

Q. 4 (a) Prove that A subring of Noetherian ring need not be Noetherian. 8

(b) Let N be a nil ideal in a Noetherian ring R. Prove that N is nilpotent. 8

UNIT-III

Q. 5 (a) State and prove maschke as theorem. 8

(b) Let D_n and D'_m be $n \times n$ and $m \times m$ matrix rings over division rings D and D' respectively, such that $D_n \simeq D'_m$ Prove that $n = m$ and $D \simeq D'$. 8

Q. 6 State and prove Noether-Lasker's theorem. 16

UNIT-IV

Q. 7 (a) If $T \in A(v)$ has all its characteristic roots in F, prove that there is a basis of V in which the matrix of T is triangular. 10

(b) If $T \in A(v)$ is nilpotent, then prove that $\alpha_0 + \alpha_1 T + \cdots + \alpha_m T^m, \alpha_i \in F$, is invertible if $\alpha_0 \neq 0$. 6

Q. 8 (a) Prove that two nilpotent linear transformations are similar if and only if they have the same invariants. 10

(b) Prove that every linear transformation $T \in A(v)$ satisfies its characteristic polynomial. Every characteristic root of T is a root of $p_T(x)$. 6

P.T.O.

UNIT-V

Q. 9 Compulsory question. (2 marks each)

i. Define completely reducible module.

ii. Define cyclic module.

iii. Define Noetherian module.

iv. Define left annihilator of a non empty set S in a ring R.

v. State (only) wedderburn – Artin theorem.

vi. Define P-primary module.

vii. When a subspace W of vector space V is called invariant under ateransformation T?

viii. Define invariants of linear transformation T.

S.No. 11010

M.Sc. (Mathematics), Second Semester, May 2017

Object Oriented Programming with C++

Time: 3 Hours Marks:60

Note: Attempt five questions in all, selecting one question from unit I to IV. Unit V is compulsory. All questions carry equal marks.

UNIT-I

Q. 1 (a) What do you mean by Object Oriented Programming? Name some object oriented language? 6

(b) What is the basic structure of a C++ program? 6

Q. 2 Write a short note on the following: 3x4=12

(a) Data types

(b) Operators

(c) Control Structures

UNIT-II

Q. 3 (a) How to pass arguments to a function? Explain with the help of example. 6

(b) What do you mean by function overloading? Explain with the help of example. 6

Q. 4 Define the following terms: 4x3=12

(a) Abstract Class

(b) Virtual Class

(c) Friend Function

(d) Pointers

UNIT-III

Q. 5 (a) Differentiate between constructor and destructor. 6

(b) What do you mean by inheritance? Define public and private inheritance levels. 6

Q. 6 Write a short note on the following: 3x4=12

(a) Derived Class

(b) Member function

(c) Polymorphism

UNIT-IV

Q. 7 (a) What are errors? How errors are handled in C++? 6

(b) What is file? What are binary files? How files are handled in C++? 6

Q. 8 (a) What are streams? Write a short note on stream classes. 6

(b) How outputs are managed using manipulators? Explain with the help of example. 6

P.T.O.

UNIT-V

Q. 9 Compulsory question. 12

i. Write any two applications of object oriented programming.

ii. What are source files?

iii. What do you mean by arrays of objects?

iv. How to return value from function?

v. What is operator overloading?

vi. What are virtual functions?

vii. What are unformatted I/O?

viii. How to open and close a file?

Sr. No. 614

M.Sc. (Mathematics), Second Semester Examination, May 2017

Complex Analysis – II

Time: 3 Hours   Max. Marks: 80

Note: Attempt one question from each Unit I to IV. Unit V is compulsory.

Unit-I

1. (a) State and prove Montel's Theorem. (8)
   (b) Define integral function and prove that the most general integral function with no zeros is of the form $e^{g(z)}$, where $g(z)$ is itself an integral function. (8)

2. (a) Prove that $\sqrt{\pi}\ \gamma(2z) = g^{2z-1}\ \gamma(z)\ \gamma(z+\frac{1}{2})$ (8)
   (b) if $< f_n >$ is a sequence in $H(G)$ and f ∈ C (G, C) such that $f_n \to f$, then prove that f is analytic and $f_n^{(k)} \to f^{(k)}$ for each integer k ≥ 1. Here H(G) denote the collection of all holomorphic functions on the open subset G of Complex plane and C (G, C) is the set of all continuous funcions from G to C. (8)

Unit -II

3. (a) Discuss analytic continuation of Riemann zeta function. (8)
   (b) State and Prove Mittag Leffler's Theorem. (8)

4. (a) If the radius of convergence of the power series
   $$f(z) = \sum_{n=0}^{\infty} a_n (z - z_0)^n$$
   is non-zero finite, then show that $f(z)$ has at least one singularity on the circle of convergence. (8)
   (b) What is analytic continuation? Prove the uniqueness of anlytic continuation along a curve. (8)

Unit -III

5. (a) State and Prove Harnack's Theorem for Harmonic functions. (8)

P.T.O.

   (b) Suppose $f(z)$ is analytic function in the ring shaped region $r_1 \leq |z| \leq r_3$. Let $r_1 < r_2 < r_3$ and let M_i be the maximum value of $|f(z)|$ on the circle $|z| = r_i, i = 1,2,3$ then prove that $M_2^{\log(\frac{r_3}{r_1})} \leq M_1^{\log(\frac{r_3}{r_2})} M_3^{\log(\frac{r_2}{r_1})}$ (8)

6. (a) Prove Poisson-Jensen formula. (8)
   (b) What do you mean by Dirichlet's Problem? Prove that this problem can be solved for the unit disk. (8)

Unit -IV

7. State and Prove Bloch's Theorem. (16)

8. (a) Suppose an analytic function f has an essential singularity at $z = a$. Prove that in each neighbourhood of a, f assumes each complex number, with one possible exception, an infinite number of times. (8)
   (b) Define Univalent function with example. Also prove that if $f(z)$ is univalent in a region D, then $f'(z) \neq 0$ in D. (8)

Unit -V

9. (8x2=16)
   (a) Show that a function which is mesomorphic in the entire z-plane is the quotient of two integral functions.
   (b) Write the domain of Convergence of Gamma function.
   (c) State Riemann's functional equation.
   (d) Prove the uniqueness of direct analytic continuation.
   (e) State Monodromy Theorem.
   (f) Define Green's function.
   (g) Find the order of exp (exp z).
   (h) Define exponent of Convergence.

S.No. 11008

M.Sc. (Mathematics), Second Semester, May 2017

Topology – II

Time: 3 Hours Marks:80

Note: Attempt five questions in all, selecting one question from each unit. Unit V is compulsory.

UNIT-I

Q. 1 (a) Give and prove one characterization of normal space. 8

(b) A topological space X is completely normal if and only if every subspace of X is normal. 8

Q. 2 (a) State and prove Urysohn Lemma. 10

(b) Show that every closed subspace of a normal space is normal. 6

UNIT-II

Q. 3 (a) Show that the intersection of any non empty family of filters on a non empty set is a filter on X. 8

(b) Prove that every filter is contained in an ultrafilter. 8

Q. 4 (a) State and prove a relationship between compactness and net convergence. 8

(b) Let E be subset of a topological space X, then show that x is a limit point of E iff there is a net $< x\gamma >$ of points of E with $x_\gamma \to x$. 8

UNIT-III

Q. 5 (a) Show that every paracompact space is normal. 8

(b) Show that metric space is paracompact. 8

Q. 6 State and prove Nagata-Smirnov Metrization theorem. 16

UNIT-IV

Q. 7 (a) Prove that a topological space is Tychonoff apace iff it is embeddable into a cube. 12

(b) Define metrizable and non-mertizable with the help of an example. 4

Q. 8 (a) show that function $f: R \to R^2$ defined by $f(x) = (x, 0)$ for each $x \in R$ is an embedding of R in R^2. 6

(b) Prove that the fundamental group of S' is isomorphic to the additive group of integers. 10

UNIT-V

Q. 9 Compulsory question. (2 marks each)

i. Define regular space and give an example of a reular space which is not T_1.

ii. State Tietze extension theorem.

iii. Define path homotopy.

iv. State stone-Cech compaitification theorem.

v. State Michaell theorem on characterization of paracompactness.

vi. Define point finite and local finiteness.

vii. State fundamental theorem of algebra.

viii. Define embedding with example.

Sr. No. 615

M.Sc. (Mathematics), Second Semester Examination, May 2017

Differential Equations – II

Time: 3 Hours Max. Marks: 80

Note: Attempt one question from each Unit I to IV. Unit V is compulsory.

Unit-I

1. (a) Write a note on self adjoint equation of second order. (8)

   (b) Discuss two basic facts and superposition principle for linear second order differential equations. (8)

2. (a) Define non-oscillatory and oscillatory functions. Give an example of each and discuss non-oscillatory and oscillatory differential equations. (8)

   (b) State and prove Abel's formula. (8)

Unit -II

3. (a) State and prove Sturm separation theorm. (8)

   (b) Consider the differential equation (8)

   $\frac{d^2u}{dt^2} + q(t)u = 0$, where $q(t)$ is a real-valued continuous, and satisfying $o < m \leq q(t) \leq M$.

   If $u = u(t) \not\equiv 0$ is a solution with pair of consecutive zeros $t = t_1, t_2 (t_1 < t_2)$, prove that $\frac{\pi}{\sqrt{M}} \leq t_2 - t_1 \leq \frac{\pi}{\sqrt{m}}$

4. (a) For autonomous systems, discuss the four kind of critical points, according to the way trajectories behave in their vicinity. (8)

   (b) Determine the nature of the critical point (0,0) of the linear system (8)

   $\frac{dx}{dt} = 2x - 7y$; $\frac{dy}{dt} = 3x - 8y$. Is the point (0,0) is stable?

Unit -III

5. (a) Find all the critical points of the nonlinear system (8)

   $\frac{dx}{dt} = 8x - y^2$; $\frac{dy}{dt} = -6y + 6x^2$ and discuss the type and stability of each of these critical points.

P.T.O.

   (b) Discuss Liapunov's direct method for stability of critical points of non-linear systems. (8)

6. (a) State and prove Bendixson's non-existence theorem. (8)

   (b) State and prove Poincare-Bendixson theorem. (8)

Unit -IV

7. (a) Find non-trivial solutions of the SLBVP (8)

   $\frac{d^2y}{dx^2} + \lambda y = 0, y(0) = 0, y(\pi) = 0$

   (b) Discuss orthogonality of characteristic functions for the SLBVP. (8)

8. (a) Prove that the eigenvalues of a SLBVP are all real. (8)

   (b) Write a note on use of implicit function theorem and fixed point theorems for periodic solution of linear and non linear differential equations. (8)

Unit -V

9. (8x2=16)

   (a) State the necessary and sufficient condition for a second order linear differential equation to be self adjoint.

   (b) State Sturm's fundamental comparison theorem.

   (c) Write down Riccatis equation.

   (d) Define stable critical point for a plane autonomous system

   (e) Define Liapunov function for a plane autonomous system.

   (f) Define limit cycle.

   (g) Define Half-path for an autonomous system.

   (h) Find the eigenvalues of the SLBVP's problem:

   $$\frac{d}{dt}\left[t\frac{du}{dt}\right] + \frac{\lambda}{t}u = 0;$$
   $$u(I) = u(e^{\pi}) = 0$$

S.No. 611

M.Sc. (Mathematics), Second Semester, May 2017

Advanced Abstract Algebra – II

Time: 3 Hours | Marks:80

Note: Attempt five questions in all, selecting one question from each unit. Unit V is compulsory.

UNIT-I

Q. 1 (a) Prove that direct product of a finite set of nilpotent groups is nilpotent. 8

(b) Prove that a nilpotent group is finite if it is generated by a finite number of elements each having finite order. 8

Q. 2 (a) Prove that Subgroup of a nilpotent group is nilpotnet. 8

(b) If the nilpotent group G has a maximal subgroup M, then prove that $M \trianglerighteq G$ and $\frac{G}{M}$ has prime order. 8

UNIT-II

Q. 3 (a) Let $T \in A(V)$ and $W \subseteq V$ is invariant under T, then T induces a linear transformation $\top$ on $\frac{v}{w}$, defined by $(v+w)\top = T(v) + w$. Then, prove that minimal polynomial of $\top$ over F divides that of T.

8

(b) Let $T \in A(V)$, then prove that there exists a subspace W of V, invariant under T, such that $V = V_1 \oplus W$, where T is nilpotent of index n_1 and $V_1 = < z, T(z), \dots\dots, T^{n_1-1}(z) >$ where $Z \in V$ such that $T^{n_1-1}(z) \neq 0$. 8

Q. 4 (a) Prove that two nilpotent L.T. are similar iff they have same invariants. 8

(b) Let $T \in A(V)$ has all its characteristic roots in F, then prove that T satisfies a polynomial of degree n. 8

UNIT-III

Q. 5 (a) If an R-module M is generated by a set $\{x_1, x_2, \dots, x_n\}$ and $| \in R$, then $M = \{r_1x_1, r_2x_2, + \dots\dots + , r_nx_n : r_i \in R\}$. 8

(b) State and prove Schur's Lemma. 8

Q. 6 (a) Let R be a ring with unity and let M be an R-module. Then the following statements are equivalent: 8

i) M is simple

ii) M ≠ {0}, and M is generated by every non-zero element of M.

iii) M≅ $R|I$, where I is a maximal left ideal of R.

P.T.O

P.T.O.

(b) Let M be a finitely generated free module over a commutative ring R. Then, all bases of M have the same number of elements. 8

UNIT-IV

Q. 7 State and prove Wedderburn-Artin Theorem. 16

Q. 8 (a) Prove that every submodule and homomorphic image of a noetherian module is noetherian. 8

(b) Let R be a noetherian ring having no nonzero nilpotent ideals. Then, prove that R has not non-zero nil ideal. 8

UNIT-V

Q. 9 Compulsory question. (2 marks each)

i. Prove that a group G is abelian iff $[x, y] = e \forall x, y \in G$.

ii. Define upper and lower central series of a group.

iii. Define cyclic subspace w.r.t. a nilpotent L.T.

iv. Porve that if $T, S \in A(V)$ are nilpotent and TS=ST, then TS is also nilpotent.

v. Define rank of a finitely generated module.

vi. Define Idempotents.

vii. Give example of a module which is noetherian but not artinian.

viii. Differentiate nil and nilpotent ideals.

M. Sc. (Mathematics)

Sr No 12068

II nd Sem. Examination, May 2017

Methods of Applied Mathematics

Time:- 3 Hours

M.M:-80

Section - I

1) a) Solve heat equation in cylinderical polar co-ordinates. (8)

b) Obtain axially-symmetrical solution of 3-D Laplace equation. (8)

2) a) Explain the concept of Gradience in curvi-Linear coordinates. (8)

b) Find divergent and curl in cylindrical coordinates (r, θ, x) where $x_1 = r\cos\theta$, $x_2 = r\sin\theta$, $x_3 = z$ (8)

Section - II

3) Obtain the solution of free vibration of a large circular elastic membrane governed by initial value problem

$$c^2\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right) = \frac{\partial^2 u}{\partial t^2}, \quad 0 < r < \infty, \; t > 0,$$

$$u(r, 0) = f(r), \quad u_t(r, 0) = g(r) \text{ for } 0 \le r < \infty$$

where $c^2 = (T/p)$ = constant, T is the tension in membrane and p is surface density. (16)

4) a) Find n^{th} order Hankel transform of $f(r) = r^n \exp(-ar^2)$. (8)

b) Define relation between fourier and hankel transform. (8)

Section - III

5) State and prove Mellin inversion theorem. (..)

6) a) Define properties of hypergeometric functions. (8)

b) Find Mellin Transform of
(i) sin (t)
(ii) e^{at} (8)

section - IV

7) State and prove Branchistochrone problem. (16)

8) a) Define Euler's equation.
b) Find the extremals of functional (8)

$$\int_{x^0}^{x^1} (y^2 - y'^2 - 2y \sin x \cdot dx$$ (8)

Section- V

9)
a) Define Hankel transform.
b) What do you understand by isoperimetric problem.
c) Write down any two operational properties of hankel transform.
d) Define boundary value problem.
e) State Laplace equation in cylindrical polar co-ordinates.
f) Write heat equation for spherical polar co-ordinates.
g) Define functionals.
h) Define calculus of variations.

S.No. 11008

M.Sc. (Mathematics), Second Semester, May 2017

Topology – II

Time: 3 Hours Marks:80

Note: Attempt five questions in all, selecting one question from each unit. Unit V is compulsory.

UNIT-I

Q. 1 (a) Give and prove one characterization of normal space. 8

(b) A topological space X is completely normal if and only if every subspace of X is normal. 8

Q. 2 (a) State and prove Urysohn Lemma. 10

(b) Show that every closed subspace of a normal space is normal. 6

UNIT-II

Q. 3 (a) Show that the intersection of any non empty family of filters on a non empty set is a filter on X. 8

(b) Prove that every filter is contained in an ultrafilter. 8

Q. 4 (a) State and prove a relationship between compactness and net convergence. 8

(b) Let E be subset of a topological space X, then show that x is a limit point of E iff there is a net $< x\gamma >$ of points of E with $x_\gamma \to x$. 8

UNIT-III

Q. 5 (a) Show that every paracompact space is normal. 8

(b) Show that metric space is paracompact. 8

Q. 6 State and prove Nagata-Smirnov Metrization theorem. 16

UNIT-IV

Q. 7 (a) Prove that a topological space is Tychonoff apace iff it is embeddable into a cube. 12

(b) Define metrizable and non-mertizable with the help of an example. 4

Q. 8 (a) show that function $f: R \to R^2$ defined by $f(x) = (x, 0)$ for each $x \in R$ is an embedding of R in R^2. 6

(b) Prove that the fundamental group of S' is isomorphic to the additive group of integers. 10

UNIT-V

Q. 9 Compulsory question. (2 marks each)

i. Define regular space and give an example of a reular space which is not T_1.

ii. State Tietze extension theorem.

iii. Define path homotopy.

iv. State stone-Cech compaitification theorem.

v. State Michaell theorem on characterization of paracompactness.

vi. Define point finite and local finiteness.

vii. State fundamental theorem of algebra.

viii. Define embedding with example.

Sr. No. 614

M.Sc. (Mathematics), Second Semester Examination, May 2017

Complex Analysis – II

Time: 3 Hours | Max. Marks: 80

Note: Attempt one question from each Unit I to IV. Unit V is compulsory.

Unit-I

1. (a) State and prove Montel's Theorem. (8)

   (b) Define integral function and prove that the most general integral function with no zeros is of the form $e^{g(z)}$, where $g(z)$ is itself an integral function. (8)

2. (a) Prove that $\sqrt{\pi}\ \gamma(2z) = g^{2z-1}\ \gamma(z)\ \gamma(z+\frac{1}{2})$ (8)

   (b) if $< f_n >$ is a sequence in $H(G)$ and f ∈ C (G, C) such that $f_n \to f$, then prove that f is analytic and $f_n^{(k)} \to f^{(k)}$ for each integer k ≥ 1. Here H(G) denote the collection of all holomorphic functions on the open subset G of Complex plane and C (G, C) is the set of all continuous funcions from G to C. (8)

Unit -II

3. (a) Discuss analytic continuation of Riemann zeta function. (8)

   (b) State and Prove Mittag Leffler's Theorem. (8)

4. (a) If the radius of convergence of the power series

   $$f(z) = \sum_{n=0}^{\infty} a_n (z - z_0)^n$$

   is non-zero finite, then show that $f(z)$ has at least one singularity on the circle of convergence. (8)

   (b) What is analytic continuation? Prove the uniqueness of anlytic continuation along a curve. (8)

Unit -III

5. (a) State and Prove Harnack's Theorem for Harmonic functions. (8)

P.T.O.

   (b) Suppose $f(z)$ is analytic function in the ring shaped region $r_1 \le |z| \le r_3$. Let $r_1 < r_2 < r_3$ and let M_i be the maximum value of $|f(z)|$ on the circle $|z| = r_i, i = 1,2,3$ then prove that $M_2^{\log(\frac{r_3}{r_1})} \le M_1^{\log(\frac{r_3}{r_2})} M_3^{\log(\frac{r_2}{r_1})}$ (8)

6. (a) Prove Poisson-Jensen formula. (8)

   (b) What do you mean by Dirichlet's Problem? Prove that this problem can be solved for the unit disk. (8)

Unit -IV

7. State and Prove Bloch's Theorem. (16)

8. (a) Suppose an analytic function f has an essential singularity at $z = a$. Prove that in each neighbourhood of a, f assumes each complex number, with one possible exception, an infinite number of times. (8)

   (b) Define Univalent function with example. Also prove that if $f(z)$ is univalent in a region D, then $f'(z) \neq 0$ in D. (8)

Unit -V

9. (8x2=16)

   (a) Show that a function which is mesomorphic in the entire z-plane is the quotient of two integral functions.

   (b) Write the domain of Convergence of Gamma function.

   (c) State Riemann's functional equation.

   (d) Prove the uniqueness of direct analytic continuation.

   (e) State Monodromy Theorem.

   (f) Define Green's function.

   (g) Find the order of exp (exp z).

   (h) Define exponent of Convergence.

S.No. 611

M.Sc. (Mathematics), Second Semester, May 2017
Advanced Abstract Algebra – II

Time: 3 Hours | Marks:80

Note: Attempt five questions in all, selecting one question from each unit. Unit V is compulsory.

UNIT-I

Q. 1 (a) Prove that direct product of a finite set of nilpotent groups is nilpotent. 8

(b) Prove that a nilpotent group is finite if it is generated by a finite number of elements each having finite order. 8

Q. 2 (a) Prove that Subgroup of a nilpotent group is nilpotnet. 8

(b) If the nilpotent group G has a maximal subgroup M, then prove that $M \trianglerighteq G$ and $\frac{G}{M}$ has prime order. 8

UNIT-II

Q. 3 (a) Let $T \in A(V)$ and $W \subseteq V$ is invariant under T, then T induces a linear transformation $\top$ *on* $\frac{v}{w}$, defined by $(v+w)\top = T(v) + w$. Then, prove that minimal polynomial of $\top$ over F divides that of T.

8

(b) Let $T \in A(V)$, then prove that there exists a subspace W of V, invariant under T, such that $V = V_1 \oplus W$, where T is nilpotent of index n_1 and $V_1 = < z, T(z), \dots\dots, T^{n_1-1}(z) >$ where $Z \in V$ such that $T^{n_1-1}(z) \neq 0$. 8

Q. 4 (a) Prove that two nilpotent L.T. are similar iff they have same invariants. 8

(b) Let $T \in A(V)$ has all its characteristic roots in F, then prove that T satisfies a polynomial of degree n. 8

UNIT-III

Q. 5 (a) If an R-module M is generated by a set $\{x_1, x_2, \dots, x_n\}$ and $| \in R$, then $M = \{r_1x_1, r_2x_2, +\dots\dots+, r_nx_n : r_i \in R\}$. 8

(b) State and prove Schur's Lemma. 8

Q. 6 (a) Let R be a ring with unity and let M be an R-module. Then the following statements are equivalent: 8

i) M is simple

ii) $M \neq \{0\}$, and M is generated by every non-zero element of M.

iii) $M \cong R|I$, where I is a maximal left ideal of R.

P.T.O

P.T.O.

(b) Let M be a finitely generated free module over a commutative ring R. Then, all bases of M have the same number of elements. 8

UNIT-IV

Q. 7 State and prove Wedderburn-Artin Theorem. 16

Q. 8 (a) Prove that every submodule and homomorphic image of a noetherian module is noetherian. 8

(b) Let R be a noetherian ring having no nonzero nilpotent ideals. Then, prove that R has not non-zero nil ideal. 8

UNIT-V

Q. 9 Compulsory question. (2 marks each)

i. Prove that a group G is abelian iff $[x, y] = e \forall x, y \in G$.

ii. Define upper and lower central series of a group.

iii. Define cyclic subspace w.r.t. a nilpotent L.T.

iv. Porve that if $T, S \in A(V)$ are nilpotent and TS=ST, then TS is also nilpotent.

v. Define rank of a finitely generated module.

vi. Define Idempotents.

vii. Give example of a module which is noetherian but not artinian.

viii. Differentiate nil and nilpotent ideals.

S.No. 9017

M.Sc. (Mathematics), Third Semester, May 2017

Partial Differential Equations and Mechanics

Time: 3 Hours | Marks:80

Note: Attempt five questions in all, selecting one question from each unit. Unit V is compulsory. All qustions carry equal marks.

UNIT-I

Q. 1 Using the method of Separation of variables, find solution of Laplace equation in two-dimensions.

Q. 2 Find the steady state temperature distribution in a thin rectangular plate bounded by the lines $x = 0, x = a, y = 0, y = b$. The edges $x = 0, x = a, y = 0$ are kept at temperature zero while the edge $y = b$ is kept at 100^{0}c.

UNIT-II

Q. 3 Obtain solution of wave equation in cylindrical co-ordinates (r, θ, z).

Q. 4 Find the solution of $\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$, subject to $u = f(x), \frac{\partial u}{\partial t} = g(x) at\ t = 0$.

UNIT-III

Q. 5 (a) The instantaneous velocities of particles at points $(a,0,0), \left(0,\frac{a}{\sqrt{3}},0\right), (0,0,2a)$ of a rigid body are $[u,0,0], [u,0,v], [u+v, -v\sqrt{3}, \frac{v}{2}]$, respectively, referred to a rectangular cartesian frame. Find the magnitutde and direction of spin of the body and the points at which the central axis cuts the $xz - plane$.

(b) Using the formula $\frac{d\vec{F}}{dt} = \frac{\partial \vec{F}}{\partial t} + \vec{w} \times \vec{F}$, derive the velocity and acceleration in cylindrical co-ordinate system of the motion of a particle.

Q. 6 (a) A circular hoop of radius 'a', rotating in a vertical plane with spin w and with its centre at rest, is in contact with a rough plane inclined at angle $\propto$. The angle of friction for the surfaces in contact also being $\propto$. Show that, if the initial slip velocity is down the plane, the hoop remains stationary for a time $aw/(g \sin \propto)$ and then the hoop rolls down the plane with acceleration $\frac{1}{2} g \sin \propto$.

(b) A uniform rod AB of mass 2m is freely jointed at B to a second rod BC of mass m. The rods lie on a smooth horizontal plane at right angles to each other and an impulse I is applied to AB at A in a direction parallel to BC. Find the initial velocity of BC and prove that the Kinetic energy generated is $\frac{5}{6}\frac{I^2}{m}$.

P.T.O.

UNIT-IV

Q. 7 (a) Find expression for kinetic energy of a rigid body rotating about a fixed point in terms of moment of Inertia and angular velocity.

(b) A square of side "a" has particles of masses m,2m,3m and 4m at its vertices. Find the principal moments of Inertia and principal directions at the centre of square.

Q. 8 (a) Show that a uniform cuboid of mass M is equinomental with marses $\frac{M}{2.4}$ at its corners and $\frac{2}{3}m$ at its centre.

(b) Show that for a 2-D mass distribution, the values of greatest and least moments of inertia about lines passing through a point O are attained along the principal axes through O.

UNIT-V

Q. 9 Compulsory question. (2 marks each)

i. Write two dimensional heat flow equation in cartesian coordinates (x,y).

ii. Formulate BVP to find the temperature distribution in bar with ends kept at zero temperature.

iii. Define vector angular velocity.

iv. Define moment of Inertia and radius of gyration.

v. State parallel axis theorem.

vi. Define impulsive motion.

vii. Write Laplace equation in cylindrical coordinates.

viii. Define general motion of a rigid body.

Sr. No. 6172-A

M.Sc. (Maths), Third Semester Examination, Dec 2016
Analytical Mechanics & Calculus of Variations

Time: 3 Hours | Max. Marks: 80

Note: Attempt any five questions in all selecting one question from each unit. Unit V is compulsory.

Unit – I

1. (a) Derive the Euler's equation for a functional with one dependent variable. 8
   (b) Find the shortest distance between the points A(1,-1,0) and B(2,1,-1) lying on the surface 15x-7y+z=22. 8
2. (a) Describe the motivating problems of calculus of variations. 8
   (b) Among all the curves lying on the sphere $x^2 + y^2 + z^2 = a^2$ and passing through the two given points (x_0, y_0, z_0) and (x_1, y_1, z_1), find the one which has the least length. 8

Unit – II

3. (a) Define virtual displacement and virtual work. Discuss basic problem of dynamics. 8
   (b) Dervie the variation of total energy equation for conservative fields. 8
4. (a) Discuss general equation of dynamics and derive Lagrange's equation of first kind. 8
   (b) Prove that the kinetic energy of a scleronomic system can be expressed as a homogeneous function of second degree in the generalized velocities. 8

Unit – III

5. (a) Derive Hamilton canonical equations. 8
   (b) State and prove Jacobi-Poisson theorem. 8
6. (a) State and prove Hamilton's principle. 8
   (b) Derive Whittaker's equations. 8

Unit – IV

7. (a) Show that the transformation $Q = \log\left(\frac{1}{q} sinp\right), P = q\ cotp$ is canonical. 8
   (b) State and prove Jacobi's theorem. 8
8. (a) Derive the Jacobian matrix of a canonical transformation. 8
   (b) Prove that Poisson brackets are invariant under canonical transformation. 8

Unit – V

9. 16

   a) Define isoperimetric problem.
   b) Define variation of a functional.
   c) Define non-holonomic constraints.
   d) Define rheonomic system.
   e) Find the relation between Lagrange action W* and Hamilton action W.
   f) Define Lagrangian and Hamiltanion variables.
   g) Define complete integrals.
   h) For Poisson bracket prove that
      i) $(\phi\ \psi) = -(\psi\ \phi)$
      ii) $(c\ \phi\ \psi) = c\ (\phi\ \psi)$

S.No. 9019

M.Sc. (Mathematics), Third Semester, May 2017
Mathematical Statistics

Time: 3 Hours Marks:80

Note: Attempt five questions in all, selecting one question from each unit. Unit V is compulsory. All questions carry equal marks.

UNIT-I

Q. 1 (a) Give Axiomatic definition of probability. What is the chance that a leap year, selected at random will contain 53 Sundays?

(b) A problem in Statistics is given to five students. Their chances of solving it are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{4}$ *and* $\frac{1}{5}$. What is the probability that the problem will be solved?

Q. 2 (a) For n events $A_1, A_2, \dots\dots, A_n$, prove that $P(\bigcap_{i=1}^{n} A_i) \geq \sum_{i=1}^{n} P(A_i) - (n-1)$.

(b) In a bolt factory, machines A,B and C manufacture respectively 25%, 35% and 40% of total of their output 5,4,2 percents are defective bolts. A bolt is drawn at random from the product and is found to be defective. What are the probabilities that it was manufactured by machines A,B and C?

UNIT-II

Q. 3 (a) Explain the following along with one example for each: Marginal and conditional distributions, probability density function, and distribution function for continuous random variables.

(b) A random variable X has the following probability function:

X:	-2	-1	0	1	2	3
f(x):	K	0.3	2K	0.2	0.1	3K

Find the value of K; P(-2<x<2); P(x<0) and Mean & variance for the distribution.

Q. 4 (a) Find the constant C such that the function

$f(x) = \begin{cases} cx^2; & 0<x<3 \\ 0 & ; otherwise \end{cases}$ is a density function and compute P(1<x<2) and P(1<x≤2): Also obtain distribution function for the random variable.

(b) State and prove Chebycher's inequality. Also define momnet generating function of a random variable about its mean & origin.

UNIT-III

Q. 5 (a) Define Poisson's distribution. Prove that mean and variance for Poisson's distribution are same.

(b) Obtain moment generating functions for Binomial and geometric distributions. Also obtian their means and variances.

P.T.O.

Q. 6 Show that for a unform distribution:

$f(x) = \frac{1}{2a}, -a < x < a$, The M.G.F. about origin is $\frac{\sinh at}{at}$. Also obtain its moments about mean.

(b) Define Normal distribution. Show that mean and mode for this distribution are same.

UNIT-IV

Q. 7 (a) What do you mean by unbiasedness and efficiency of an estimator?

(b) Explain the following: Null and Alternate Hypothesis; Critical region and level of significance.

Q. 8 (a) Twenty people were attacked by a disease and only 18 survived. Will you reject the hypothesis that the survival rate, if attacked by this disease, is 85% in favour of the hypothesis that it is more, at 5% level. (Use Large Sample Test).

(b) The means of two single large samples of 1000 and 2000 members are 67.5 inches and 68.0 inches respectively. Can the samples be regarded as drawn from the same population of standard deviation 2.5 inches?

UNIT-V

Q. 9 Compulsory question.

i. Define: Trial and event; exhaustive events, and Independent events.

ii. Define conditional probability.

iii. Explain mathematical expectation of a random variable.

iv. Obtain second moment abot mean for Normal distribution.

v. Show than mean≥variance for a Binomial distribution.

vi. Write density function for exponential distribution and obtain its first moment about origin.

vii. Define Point estimation.

viii. Define type-I & type-II errors.

S.No. 9017

M.Sc. (Mathematics), Third Semester, May 2017

Partial Differential Equations and Mechanics

Time: 3 Hours Marks:80

Note: Attempt five questions in all, selecting one question from each unit. Unit V is compulsory. All qustions carry equal marks.

UNIT-I

Q. 1 Using the method of Separation of variables, find solution of Laplace equation in two-dimensions.

Q. 2 Find the steady state temperature distribution in a thin rectangular plate bounded by the lines $x = 0, x = a, y = 0, y = b$. The edges $x = 0, x = a, y = 0$ are kept at temperature zero while the edge $y = b$ is kept at 100^0c.

UNIT-II

Q. 3 Obtain solution of wave equation in cylindrical co-ordinates (r, θ, z).

Q. 4 Find the solution of $\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$, subject to $u = f(x), \frac{\partial u}{\partial t} = g(x) at\ t = 0$.

UNIT-III

Q. 5 (a) The instantaneous velocities of particles at points $(a, 0,0), \left(0, \frac{a}{\sqrt{3}}, 0\right), (0,0,2a)$ of a rigid body are $[u, 0,0], [u, 0, v], [u + v, -v\sqrt{3}, \frac{v}{2}]$, respectively, referred to a rectangular cartesian frame. Find the magnitutde and direction of spin of the body and the points at which the central axis cuts the $xz - plane$.

(b) Using the formula $\frac{d\vec{F}}{dt} = \frac{\partial \vec{F}}{\partial t} + \vec{w} \times \vec{F}$, derive the velocity and acceleration in cylindrical co-ordinate system of the motion of a particle.

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P.T.O.

UNIT-IV

Q. 7 (a) Find expression for kinetic energy of a rigid body rotating about a fixed point in terms of moment of Inertia and angular velocity.

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i. Write two dimensional heat flow equation in cartesian coordinates (x,y).

ii. Formulate BVP to find the temperature distribution in bar with ends kept at zero temperature.

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v. State parallel axis theorem.

vi. Define impulsive motion.

vii. Write Laplace equation in cylindrical coordinates.

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Roll No. ....

Sr. No. 13021

M.Sc. (Mathematics), 3rd Semester, June 2017
Integral Equations and Calculus of Variations
Paper : MM-505 Opt. (i)

Time: 3 Hours

Marks: 80

Note: Attempt five questions in all, selecting one question from each unit. Unit V is compulsory.

Unit I

1. a) Form an integral equation corresponding to the IVP

$$y''' + xy'' + (x^2 - x)y = xe^x + 1;$$
$$y(0) = 1 = y'(0), \quad y''(0) = 0 \qquad (8)$$

b) Find the Neumann Series for the solution of the integral equation

$$y(x) = 1 + x + \lambda \int_0^x (x - t) y(t) dt \qquad (8)$$

2. a) Solve the following integral equation by method of successive approximation

$$y(x) = f(x) + \lambda \int_0^x K(x,t) y(t) dt \qquad (8)$$

b) Solve the integral equation

$$y(x) = f(x) + \lambda \int_0^x J_0(x - t) y(t) dt \qquad (8)$$

Unit II

3. a) Reduce the following boundary value problem into an integral equation :

$$d^2y/dx^2 + \lambda y = 0 \quad with \ y(0) = 0, \quad y(l) = 0 \qquad (8)$$

b) Find the Resolvent kernel by using the Fredholm determinants

$$y(x) = f(x) + \lambda \int_{-1}^{1} (xt - x^2 t^2) y(t) dt \qquad (8)$$

4. a) Solve the integral equation and discuss all possible cases with the method of degenerate kernels

$$y(x) = f(x) + \lambda \int_0^1 (1 - 3xt) y(t) dt \qquad (10)$$

b) By considering only the first two terms of e^{xt}, find the approximate solution to the Fredholm integral equation

$$y(x) = x + \int_{-1}^{1} e^{xt} y(t) dt \qquad (6)$$

Unit III

5. a) Using Green's function, solve the boundary value problem

$$d^2y/dx^2 - y = x \quad with \; y(0) = y(1) = 0$$ (8)

b) Solve $y'' + x = 0; \quad y(0) = 0, \; y(1) = 0$, using Green's function and verify the answer. (8)

6. a) Using Hilbert-Schmidt theorem, solve the following integral equation

$$y(x) = (x+1)^2 + \int_{-1}^{1} (xt + x^2t^2) y(t) dt$$ (8)

b) Show that the eigen function of a symmetric kernel corresponding to different eigen values are orthogonal. (8)

Unit IV

7. a) Find the general solution of Euler's equation associated with functional

$$I[y] = \int_a^b \sqrt{x(1+y'^2)}dx$$ (8)

b) State and give the solution of Brachistochrone problem. Justify your answer. (8)

8. a) Obtain geodesics on the surface of sphere. (8)

b) Find the extremal of the functional

$$I[y] = \int_0^{\pi} (y'^2 - y^2)dx$$

under the condition $y(0) = 0, \; y(\pi) = 1$ and subject to the constraint

$$\int_0^{\pi} y dx = 1$$ (8)

Unit V

9. (i) Define integral equation.
(ii) Classify integral equations.
(iii) Define separable or degenerate kernel with example.
(iv) Give four properties of Green's function.
(v) Prove that :

$$\|K\| \le \left[\iint |K(x,t)|^2 \, dxdt \right]^{1/2}$$

(vi) Show that eigenvalues of a symmetric kernel are real.
(vii) Define isoperimetric problems with suitable example.
(viii) Define natural boundary conditions.

(2×8=16)

S.No. 9019

M.Sc. (Mathematics), Third Semester, May 2017

Mathematical Statistics

Time: 3 Hours Marks:80

Note: Attempt five questions in all, selecting one question from each unit. Unit V is compulsory. All questions carry equal marks.

UNIT-I

Q. 1 (a) Give Axiomatic definition of probability. What is the chance that a leap year, selected at random will contain 53 Sundays?

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$f(x) = \begin{cases} cx^2; & 0<x<3 \\ 0 & ; otherwise \end{cases}$ is a density function and compute P(1<x<2) and P(1<x≤2): Also obtain distribution function for the random variable.

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UNIT-IV

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(b) Explain the following: Null and Alternate Hypothesis; Critical region and level of significance.

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Roll No. ....

Sr. No. 13021

M.Sc. (Mathematics), 3rd Semester, June 2017

Integral Equations and Calculus of Variations

Paper : MM-505 Opt. (i)

Time: 3 Hours | Marks: 80

Note: Attempt five questions in all, selecting one question from each unit. Unit V is compulsory.

Unit I

1. a) Form an integral equation corresponding to the IVP

$$y''' + xy'' + (x^2 - x)y = xe^x + 1;$$
$$y(0) = 1 = y'(0), \quad y''(0) = 0 \qquad (8)$$

b) Find the Neumann Series for the solution of the integral equation

$$y(x) = 1 + x + \lambda \int_0^x (x-t)y(t)dt \qquad (8)$$

2. a) Solve the following integral equation by method of successive approximation

$$y(x) = f(x) + \lambda \int_0^x K(x,t)y(t)dt \qquad (8)$$

b) Solve the integral equation

$$y(x) = f(x) + \lambda \int_0^x J_0(x-t)y(t)dt \qquad (8)$$

Unit II

3. a) Reduce the following boundary value problem into an integral equation :

$$d^2y/dx^2 + \lambda y = 0 \text{ with } y(0) = 0, \quad y(l) = 0 \qquad (8)$$

b) Find the Resolvent kernel by using the Fredholm determinants

$$y(x) = f(x) + \lambda \int_{-1}^1 (xt - x^2t^2)y(t)dt \qquad (8)$$

4. a) Solve the integral equation and discuss all possible cases with the method of degenerate kernels

$$y(x) = f(x) + \lambda \int_0^1 (1-3xt)y(t)dt \qquad (10)$$

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$$y(x) = x + \int_{-1}^1 e^{xt}y(t)dt \qquad (6)$$

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b) Solve $y'' + x = 0;\quad y(0) = 0,\ y(1) = 0$, using Green's function and verify the answer. (8)

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$$I[y] = \int_a^b \sqrt{x(1+y'^2)}dx$$ (8)

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9. (i) Define integral equation.
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(2×8=16)